Math 2263, Quiz 4
You must show all work for full credit, you have 15 min to finish it.

1.(5 pt) If $x^3 + e^y + \sin(z) = 0$, find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$.
Solution: Let $f(x, y, z) = x^3 + e^y + \sin(z)$, then by chain rule we know $\frac{\partial z}{\partial x} = -\frac{f_x}{f_z} = -\frac{3x^2}{\cos(z)}$ and $\frac{\partial z}{\partial y} = -\frac{f_y}{f_z} = -\frac{e^y}{\cos(z)}$

2.(5 pt) Use the chain rule to find $\frac{dz}{dt}$ where $z = x^2y$, $x = \cos(t)$, $y = e^t$.
Solution: By chain rule, we know $\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt} = 2xy(-\sin(t)) + x^2(e^t) = (\cos(t))^2e^t - 2\sin(t)\cos(t)e^t$.

3.(5 pt) Find the parametric equation of the tangent line to the curve of intersection of quadratic surfaces $x^2 + y^2 + z^2 = 5$ and $z = 2x^2 + y^2$ at the point $(1, 0, 2)$.
Solution: The normal vectors of two tangent planes are $(2x, 2y, 2z) \mid_{(1,0,2)} = (2, 0, 4)$ and $(-4x, -2y, 1) \mid_{(1,0,2)} = (-4, 0, 1)$. So the directional vector of the tangent line is just the cross product of two normal vectors which equals to $(0, -18, 0)$. So the parametric equation of the tangent line is $x = 1, y = -18t, z = 2$.